**Homework 3** Due 18:00, October 20, 2021

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# Problem 3.1

****Prove by Well Ordering Principle:

**Proof by WOP:** ;

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, so :

# Problem 3.2

Prove by Principle of Mathematical Induction:

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**Proof by MI:**

*, so we can add*

*+*

Therefore, *we proved that if P(n) is true then P(n+1) is also true.*

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# Problem 3.3

Give an inductive proof that the Fibonacci numbers *Fn* and *Fn*+1 are relatively prime for all *n* ≥ 0. The Fibonacci numbers are defined as follows:

*F*0 = 1*, F*1 = 1*, Fn* = *Fn−*1 + *Fn−*2*,* ∀*n* ≥ 2

**Proof by Induction:**

Therefore, *we proved that if P(n) is true then P(n+1) is also true.*

# Problem 3.4

Prove by induction that for all integers *n* ≥ 2

4*n* + 7 ≤ 5*n.*

**Proof by Induction:**

Therefore, *we proved that if P(n) is true then P(n+1) is also true.*

# Problem 3.5

Find the flaw in the following bogus proof that *an* = 1 for all nonnegative integers *n*, whenever *a* is a nonzero real number.

**Proof**. The bogus proof is by induction on *n*, with hypothesis

*P* (*n*) : ∀*k* ≤ *n ak* = 1

where *k* is a nonnegative integer valued variable.

**Base Case**: *P* (0) is equivalent to *a*0 = 1, which is true by definition of *a*0, (by convention, this holds even if *a* = 0).

**Inductive Step**: By induction hypothesis, *ak* = 1 for all *k* ∈ N such that *k* ≤ *n*. But then

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which implies that *P* (*n* + 1) holds. It follows by induction that *P* (*n*) holds for all *n* ∈ N, and in particular,

*an* = 1 holds for all *n* ∈ N.

as our hypothesis states that

Therefore, *We 𝑐𝑎𝑛𝑛𝑜𝑡 𝑐𝑜𝑛𝑡𝑖𝑛𝑢𝑒 𝑜𝑢𝑟 𝑖𝑛𝑑𝑢𝑐𝑡𝑖𝑜𝑛 𝑝𝑟𝑜𝑜𝑓 𝑏𝑎𝑠𝑒𝑑 𝑜𝑛 𝑡ℎ𝑖𝑠*

*𝑐𝑜𝑛𝑡𝑟𝑎𝑑𝑖𝑐𝑡𝑖𝑜n.*

# Problem 3.6

Let *BM* be the set of all sequences (or strings) of square brackets [ and ] that are matched (left with right). For example, the following three strings are in *BM* :

[ ] [ ] and [ [ ] ] and [ [ [ ] [ ] ] ]*,*

but these strings are not in *BM* :

[ [ ] ] ] and ] [ [ ] ]*.*

Recursively define the set BM of matched brackets as follows:

**Base case.** *λ* ∈ *BM* (empty string is in *BM* ).

**Constructor case.** If *s, t* ∈ *BM* , then [ *s* ] *t* ∈ *BM* .

Prove by structural induction that (recursively-defined) matched strings from *BM* always have an equal number of left and right brackets.

***Prove by structural induction – P(***#[(s) = #](s)ю

#[() = 0 #]()

#[ ([s]t) =

= #[ ( [ ) + #[ ( s ) + #[ ( ] ) + #[ ( t )

= 1 + #[ ( s ) + 0 + #[ ( t ) /\*def #[ ( ) \*/

= 1 + #] ( s ) + 0 + #] ( t ) /\* by P(s) and P (t)\*/

= 0 + #] ( s ) + 1 + #] ( t )

= #] ( [ ) + #] ( s ) + #] ( ] ) + #] ( t ) /\*def #] ( ) \*/

= #] ([s]t)

Therefore by structural induction that P (s) holds for all s ∈ *BM*

# Problem 3.7

A robot Romeo moves on two-dimensional integer grid. It starts out at (0*,* 0) and is allowed to move in any of these four ways:

**1.** (+2*,* 1): right 2, down 1; **2.** ( 2*,* +1): left 2, up 1; **3.** (+1*,* +3); **4.** ( 1*,* 3).

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Julietta waits him at (1*,* 1). Prove that Romeo never will be able to reach his love.

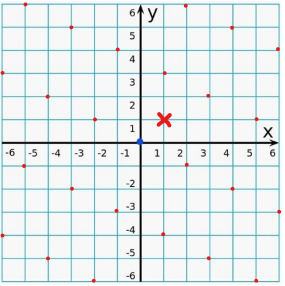
**Proof by Induction:**

, and we get the following system

*any position ⟨a,b⟩, let r(a,b)=(3a−b)mod7, the remainder when 3a−b is divided by 7. If ⟨x,y⟩ is one of the legal moves , then r(a+x,b+y)= (3(a+x)−(b+y))mod7= ((3a−b)+(3x−y))mod7= (3a−b)mod7= r(a,b), since (3x−y)mod7=0.*

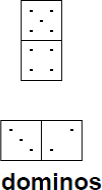
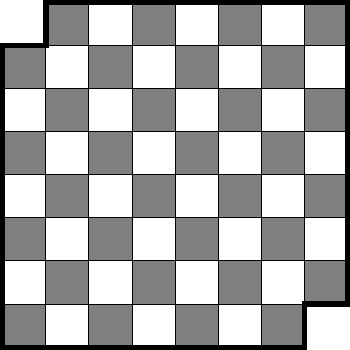
*Thus if the robot starts at ⟨0,0⟩, then r(a,b)=r(0,0)=0 for each position ⟨a,b⟩ (that the robot can move to)*

Therefore, *𝑆𝑖𝑛𝑐𝑒 𝑥, 𝑦 ∉ ℤ, 𝑅𝑜𝑚𝑒o robot cannot reach ⟨1,1⟩*



# Problem 3.8

Can you tile an 8×8 chessboard with 31 dominos if opposite corners are removed? Argue your answer rigourously.



***The Prove:***

Therefore, *we can’t tile an 8×8 chessboard with 31 dominos if opposite corners are removed*

# Problem 3.9

## BONUS PROBLEM

Elementary functions *EF* are the set of functions of one real variable defined recursively as follows:

## Base Cases.

* Identity function, *id*(*x*) = *x* is in *EF* .
* Any constant function is in *EF* .
* The sine function sin(*x*) is in *EF*

**Constructor Cases.** If *f, g* ∈ *EF* , then so are

* *f* + *g*, *fg*, 2*g*;
* The inverse function *f−*1;
* The composition *f* ◦ *g*.

Prove the following:

Therefore, *we proved that*

Therefore, *we proved that*

Therefore, *we proved that*